

3-Form Flux Compactification of Salam-Sezgin Supergravity

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Abstract

The compactification of 6 dimensional Salam-Sezgin model in the presence of 3-form flux H is investigated. We find a torus topology for this compactification with two cusps which are the places of branes, while at the limit of large size L of the compact direction we also obtain sphere topology. This resembles the Randall-Sundrum I,II model. The branes at one of the cusps can be chosen to be 3- and 4-branes which fill our 4-dimensional space together with the fact that $H = 0$ at this position restores the Lorentz symmetry. This compactification also provides an example for the so-called ‘time warp’ solution, [0812.5107 [hep-th]]. According to a no-go theorem in $d \neq 6$, the time warp compactification violates the null energy condition. While the theorem is quiet for $d = 6$, our model gives a time warp compactification which satisfies the null energy condition. We also derive the four dimensional effective Planck mass which is not obvious due to the time warp nature of the solution.

1 Introduction

In more than a decade, since the celebrated work of Randall-Sundrum [1, 2], the warp compactification, has been considered as a new approach to explain the hierarchy problem in 4-dimensional space-time as a low energy limit of higher dimensional theories. This approach brought new phenomenological results with more hopes to find evidences for higher dimensional theories in a foreseeable future.

Long before the warp compactification idea, the six dimensional gauged supergravity was studied by Salam-Sezgin in [3–6], as a simple model to obtain the supersymmetric vacua by compactification to 4 dimensions. It has also interesting applications in cosmological model building [7–10]. On the other hand, in another development [11], it has been shown that this model can be derived from the string theory which strengthens its importance as a descendent of a fundamental theory. In a modern view, the Salam-Sezgin supergravity is rich enough, while simple, to provide the warp compactification including fluxes [12–18]. In [17], it was found that four dimensional Minkowski space solution is not only possible, but inevitable if one requires maximal symmetry in four dimensions and compactness of internal space. Based on these features, it is worth to work out its various warp compactifications.

The bosonic part of the model contains the metric, dilaton, a 2-form $F_{(2)}$ and a 3-form $H_{(3)}$ as field strengths. In [12–19] a static warped solution has been found for $H = 0$ and $F \neq 0$. A dynamical model was proposed in [20]. For some recent developments see [21–23]

In all of the case, so far H has been set to zero. Beside technical reasons which make equations hard to solve when H is included, it is obvious that the presence of a 3-form in a 6-dimensional space can not support a symmetric 4-dimensional compactification. Nonetheless, we will see soon that the situation is not a disaster and one may find an appropriate interpretation.

In this paper, we have considered a 4-dimensional compactification with H field which is extended along the 2-dimensional internal space and the time direction. This kind of discrimination between time and other non-compact spatial directions, may suggest its application to cosmological models, however, here we restrict ourselves to a static model and postpone the study of dynamical solutions to future. Should we need a warp compactification, H field configuration suggests the warp factors in time and spatial directions should be different. This is what has been called ‘time warp’ recently in [24]. The ratio of time and spatial warp factor is the light speed which depends on the internal coordinate by construction. There is a no-go argument in [24] according to which the internal space in time warped solutions can not be compact, unless the null energy condition is violated. Meanwhile the validity of this no-go theorem in $d = 6$ is under query, and indeed our model provides a counterexample in which the null energy condition can be satisfied even for the compact case.

We show that it is needed to solve equations in different patches and join them by Israel junction conditions [25]. These conditions could be satisfied only when one introduces the branes at joining positions [26]. In this way we find out branes sitting at the middle and two ends of the compact space. More explicitly, we consider a compact internal

space with axial symmetry which satisfies equations of motion in the interval $[0, L]$ for the radial coordinate, z , and then extended to $[-L, 0]$ interval with L and $-L$ identified. Thus we have a torus topology, with two cusps at 0 and L which are the positions of branes. We consider minimal number of branes and show that it is possible to introduce 3- and 4-branes filling our 4-dimensional space where the 4-brane wrapped and 3-branes are smeared over the internal circle [27]. On the other side at $z = L$, in addition to 3- and 4-branes, we need 0-branes to satisfy the junction conditions with time-space asymmetry. These 0-branes smeared over the world volume of the 4-brane. This configuration makes it possible to have a 4-dimensional symmetric space at $z = 0$. To ensure about this symmetry we need to consider the behavior of H field at $z = 0$. Indeed H is discontinuous at this position, since branes act as a surface of polarized charges for the electrical H field, so the H field changes the sign while crossing the brane. The mean value of H would be zero at $z = 0$ which together with the branes configuration restore the 4-dimensional lorentz symmetry at $z = 0$.

At the first look, it may seem impossible to introduce an effective covariant 4-dimensional gravity, however, a fine tuning of the parameters make it possible to obtain the effective Planck mass and 4-dimensional symmetry in the linear approximation.

We organize the paper as follows. In the next section, equations of motion including the metric, dilaton and H field are solved. In section 3, we introduce the junction conditions and branes. These conditions also fix some of the integration constants and we discuss the domain of independent parameters. The section 4 is devoted to discuss the large L limit where depending on the parameters, the internal azimuthal radius may diverge or shrink at large L to give new topologies. In section 5, we show the validity of the null energy condition. In section 6, the effective four dimensional gravity is considered and the effective Planck mass is derived. We conclude in section 7.

2 Equations of motion and H -flux solution

Let us start with the bosonic part of generalized Salam-Sezgin model with the following Lagrangian [3–6]:

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2\kappa^2} (-\mathcal{R} - \partial_M \phi \partial^M \phi) - \frac{1}{4} e^{-\phi} F_{MN} F^{MN} - \frac{1}{6} e^{-2\phi} H_{MNP} H^{MNP} - \frac{2g^2}{\kappa^4} e^\phi \quad (2.1)$$

where capital latin indices are six dimensional indices, ϕ is the dilaton, F and H are 2 and 3-form fields. Equations of motion follows as,

$$\begin{aligned} -\mathcal{R}_{MN} &= \partial_M \phi \partial_N \phi + \kappa^2 e^{-\phi} \left(F_{MN}^2 - \frac{1}{8} F^2 G_{MN} \right) \\ &\quad + \frac{1}{2} \kappa^2 e^{-2\phi} \left(H_{MN}^2 - \frac{1}{6} H^2 G_{MN} \right) + \frac{g^2}{\kappa^2} e^\phi G_{MN} \end{aligned}$$

$$\begin{aligned}
& \square\phi + \frac{\kappa^2}{6}e^{-2\phi}H_{MNP}H^{MNP} + \frac{\kappa^2}{4}e^{-\phi}F_{MN}F^{MN} - \frac{2g^2}{\kappa^2}e^\phi = 0 \\
& D_M(e^{-2\phi}H^{MNP}) = 0 \\
& D_M(e^{-\phi}F^{MN}) + e^{-2\phi}H^{MNP}F_{MP} = 0
\end{aligned} \tag{2.2}$$

To solve the above equations, we consider compactification to 4-dimension with axial symmetry in the internal space. Since we are looking for static solutions, we take all fields to be dependent on the internal radial coordinate η as in the following ansatze,

$$\begin{aligned}
ds^2 &= -e^{2w(\eta)}dt^2 + e^{2a(\eta)}\delta_{ij}dx^i dx^j + e^{2v(\eta)}d\eta^2 + e^{2b(\eta)}d\theta^2 \\
F &= 0, \quad e^\phi = e^{\phi(\eta)}, \quad H = h'(\eta)dt \wedge d\theta \wedge d\eta.
\end{aligned} \tag{2.3}$$

For dimensional convenience we assume θ has length of dimension with $0 \leq \theta \leq L_\theta$. since H extensions distinguish the time from other spatial noncompact coordinates, we have included two different warp factors e^{2w} and e^{2a} in the metric. Now the equations read as,

$$(\text{Maxwell}) \quad h'' + (3a' - w' - v' - b' - 2\phi')h' = 0 \tag{2.4}$$

$$(\text{Dilaton}) \quad \phi'' + (3a' + w' - v' + b')\phi' - \kappa^2 h'^2 e^{-2(w+b+\phi)} - \frac{2g^2}{\kappa^2}e^{2v+b} = 0 \tag{2.5}$$

$$\begin{aligned}
(\text{tt Einstein}) \quad & w'' + (w' + 3a' - v' + b')w' - \frac{\kappa^2 h'^2}{2}e^{-2(w+b+\phi)} + \frac{g^2}{\kappa^2}e^{2v+\phi} = 0 \\
(\text{ii Einstein}) \quad & a'' + (w' + 3a' - v' + b')a' + \frac{\kappa^2 h'^2}{2}e^{-2(w+b+\phi)} + \frac{g^2}{\kappa^2}e^{2v+\phi} = 0 \\
(\theta\theta \text{ Einstein}) \quad & b'' + (w' + 3a' - v' + b')b' - \frac{\kappa^2 h'^2}{2}e^{-2(w+b+\phi)} + \frac{g^2}{\kappa^2}e^{2v+\phi} = 0 \\
(\eta\eta \text{ Einstein}) \quad & w'' + 3a'' + b'' + w'^2 + 3a'^2 + b'^2 + \phi'^2 - (w' + 3a' + b')v' \\
& - \frac{\kappa^2 h'^2}{2}e^{-2(w+b+\phi)} + \frac{g^2}{\kappa^2}e^{2v+\phi} = 0
\end{aligned} \tag{2.6}$$

To solve these equations we can use the gauge freedom in choosing coordinate η such that,

$$(w' + 3a' - v' + b') = 0 \tag{2.7}$$

Then suitable combinations of (2.4)-(2.6) give,

$$\begin{aligned}
h'(\eta) &= \pm q e^{2x} \\
w(\eta) &= \frac{y+x}{4} + (2\lambda_3 + \lambda_4)\eta \\
a(\eta) &= \frac{y-x}{4} + \left(\frac{-\lambda_3}{3}\right)\eta \\
v(\eta) &= \frac{5y-x}{4} + \lambda_3\eta \\
b(\eta) &= \frac{y+x}{4} - \lambda_4\eta \\
\phi(\eta) &= \frac{x-y}{2} - 2\lambda_3\eta
\end{aligned} \tag{2.8}$$

with q a real positive number and $x(\eta)$ and $y(\eta)$ can be found from,

$$\begin{aligned}
x'^2 - 2\kappa^2 q^2 e^{2x} &= \lambda_1^2 \\
y'^2 + \frac{4g^2}{\kappa^2} e^{2y} &= \lambda_2^2
\end{aligned} \tag{2.9}$$

and λ_i 's are integration constants which are not independent and satisfy,

$$\lambda_2^2 = \lambda_1^2 + 2(\lambda_3 + \lambda_4)^2 + \frac{16}{3}\lambda_3^2 \tag{2.10}$$

The general solutions of these equations are:

$$\begin{aligned}
e^{-x} &= \frac{\sqrt{2}\kappa q}{\lambda_1} f(\lambda_1(\eta - \eta_1)) \\
e^{-y} &= \frac{2g}{\kappa\lambda_2} \cosh(\lambda_2(\eta - \eta_2)) \\
f(\eta) &= \begin{cases} \pm \sinh(\eta) & \lambda_1^2 > 0 \\ \pm \eta & \lambda_1^2 = 0 \\ \pm \sin(\eta) & \lambda_1^2 < 0 \end{cases}
\end{aligned} \tag{2.11}$$

λ_2 is positive, since g , κ , e^{-y} are non-negative values. To ensure that e^{-x} for all η is a non-zero positive real number we can construct its solution as:

$$e^{-x} = \begin{cases} \frac{\sqrt{2}\kappa q}{\lambda_1} f(\lambda_1(\eta - (\eta_1 - \varepsilon))) & \eta > \eta_1 \\ -\frac{\sqrt{2}\kappa q}{\lambda_1} f(\lambda_1(\eta - (\eta_1 + \varepsilon))) & \eta < \eta_1 \end{cases}$$

where $\varepsilon > 0$. If we change the coordinate as follows,

$$\begin{aligned}
z &= \lambda_2(\eta - \eta_1) \\
z_1 &= \lambda_2\varepsilon \\
z_2 &= \lambda_2(\eta_2 - \eta_1) \\
\lambda &= \frac{\lambda_1}{\lambda_2}
\end{aligned}$$

and use the absolute value, we can construct an even solution with respect to $z = 0$. So we find

$$\begin{aligned} e^{-x} &= \frac{\kappa \tilde{q}}{\lambda} f(\lambda(|z| + z_1)) \\ e^{-y} &= \frac{\tilde{g}}{\kappa} \cosh(|z| - z_2) \end{aligned} \quad (2.12)$$

where

$$\tilde{q} = \frac{\sqrt{2}q}{\lambda_2} \quad , \quad \tilde{g} = \frac{2g}{\lambda_2}. \quad (2.13)$$

The constraint (2.10) can be written as:

$$1 = \lambda^2 + 2(\tilde{\lambda}_3 + \tilde{\lambda}_4)^2 + \frac{16}{3}\tilde{\lambda}_3^2 \quad (2.14)$$

where

$$\tilde{\lambda}_3 = \frac{\lambda_3}{\lambda_2} \quad , \quad \tilde{\lambda}_4 = \frac{\lambda_4}{\lambda_2} \quad , \quad \lambda = \frac{\lambda_1}{\lambda_2}.$$

So far we have derived general solutions to the equations of motion including integration constants. To fix these constants, we need appropriate boundary conditions or physically interesting special cases. We deal with these conditions in the following sections.

3 Branes and Israel junction conditions

In this section we study the global aspects of the above solution. Firstly as stated below (2.11), we should keep the exponential functions in the metric to be positive everywhere and this indicates that the above solutions can not be valid globally, we need to cut and join them in different patches appropriately. This has already been done at $z = 0$. Also trying to find a compact internal space, we take the z direction to be compact in some interval $[-L, L]$ with periodic boundary conditions*. We will study the noncompact limit ($L \rightarrow \infty$) later. Indeed the solution set in the previous section is valid for each segment of $(-L, 0)$ and $(0, L)$. Thus we only need to match different patches by Israel junction

*The Euler character can be calculated,

$$\chi = \frac{1}{4\pi} \int_Y \sqrt{g} R^{(2)} d^2y + \frac{1}{2\pi} \int_{\partial Y} K ds$$

where $K = g^{\theta\theta} K_{\theta\theta}$ is the geodesic curvature on the boundaries. Then,

$$\chi = 2 \frac{L_\theta}{2\pi} \left[\int_0^L e^{b-v} (b'' + b'^2 - b'v') dz - b'e^{b-v}|_0 + b'e^{b-v}|_L \right] = 0$$

This shows that the internal space is generically a torus. The Large L limit may cause a cycle shrinks as can be seen in cases d, f and h of figure 3 .

conditions. We know that these conditions ensure the continuity of the solutions and relate the derivative discontinuities to possible brane tensions. So we expect there might be some branes sitting at $z = 0$ and/or $z = L$.

The Israel junction conditions relate the jump in the derivatives of the metric to the branes tension sitting at $z = z_0$ as follows,

$$[K_{mn} - K\hat{g}_{mn}]_{z_0} + \kappa^2 t_{mn} = 0 \quad (3.1)$$

where $[f(z)]_{z_0}$ means

$$[f(z)]_{z_0} := \lim_{\epsilon \rightarrow 0^+} (f(z_0 + \epsilon) - f(z_0 - \epsilon))$$

and K_{mn} is the extrinsic curvature of constant proper radius ρ which is introduced in the following form of the metric:

$$ds^2 = d\rho^2 + \hat{g}_{mn}dx^m dx^n. \quad (3.2)$$

Then the extrinsic curvature is $K_{mn} = \frac{1}{2}\partial_\rho \hat{g}_{mn}$. The brane stress energy t^{mn} is given by

$$t^{mn} \equiv \frac{2}{\sqrt{-\hat{g}}} \frac{\delta S_{brane}}{\delta \hat{g}_{mn}} \quad (3.3)$$

In our case because the 4D maximal symmetry has been broken out, it is impossible to interpret the 4-brane stress tensor as being due to a pure tension. But we can be hopeful to find it at least along one of the branes at e.g. $z = 0$:

$$\begin{aligned} t_{\mu\nu} &= \lambda_2 T \hat{g}_{\mu\nu} \\ t_{\theta\theta} &= \lambda_2 T_4 \hat{g}_{\theta\theta} \end{aligned} \quad (3.4)$$

where $T = T_4 + \tilde{T}_3$ with $\tilde{T}_3 = \frac{T_3}{L_\theta}$. λ_2 is inserted for later convenience. These are the configuration of the stress energy tensors of a four-brane wrapping the internal circle and a three-brane which is smeared over the internal circle. This situation can't be satisfied for the other side at $z = L$ simultaneously, so in the most general form, the stress energy tensors at $z = L$ is taken to be:

$$\begin{aligned} t_{00} &= \lambda_2 (\tilde{T}_{L0} + T_{L4} + \tilde{T}_{L3}) \hat{g}_{00} \\ t_{ij} &= \lambda_2 (T_{L4} + \tilde{T}_{L3}) \hat{g}_{ij} \\ t_{\theta\theta} &= \lambda_2 T_{L4} \hat{g}_{\theta\theta} \end{aligned} \quad (3.5)$$

where in addition to 3 and 4-branes, we have considered 0-branes at L smeared over all spatial direction except for z direction. The *tilde* over the tensions shows they are the density of smeared tensions, i.e., $\tilde{T}_{L0} = T_{L0}/Vol_4$ and $\tilde{T}_{L3} = T_{L3}/L_\theta$.

Plugging our solution to the junction conditions (3.1), and after appropriate combinations, we obtain the following conditions at $z = 0, L$:

$$\begin{aligned}
[a'(z) - w'(z)]_{z=0} &= 0 \\
[b'(z) - a'(z)]_{z=0} &= \kappa^2 e^{v(0)} \tilde{T}_3 \\
[3a'(z) + w'(z)]_{z=0} &= \kappa^2 e^{v(0)} T_4 \\
[a'(z) - w'(z)]_{z=L} &= \kappa^2 e^{v(L)} \tilde{T}_{L0} \\
[b'(z) - a'(z)]_{z=L} &= \kappa^2 e^{v(L)} \tilde{T}_{L3} \\
[3a'(z) + w'(z)]_{z=L} &= \kappa^2 e^{v(L)} T_{L4} .
\end{aligned} \tag{3.6}$$

Let us consider the above conditions on the *sinh* solution. The *sine* and *linear* solutions can be derived by taking $\lambda \rightarrow i\lambda$ and $\lambda \rightarrow 0$, respectively. Firstly, write the solution as,

$$\begin{aligned}
e^{-x(z)} &= \frac{\kappa \tilde{q}}{\lambda} \sinh(\lambda(|z| + z_1)) \theta(L - |z|) \\
e^{-y(z)} &= \frac{\tilde{g}}{\kappa} \cosh(|z| - z_2) \theta(L - |z|) \\
w(z) &= \frac{y + x}{4} + (2\tilde{\lambda}_3 + \tilde{\lambda}_4)(|z| + z_3) \theta(L - |z|) \\
a(z) &= \frac{y - x}{4} - \frac{\tilde{\lambda}_3}{3}(|z| + z_3) \theta(L - |z|) \\
v(z) &= \frac{5y - x}{4} + \tilde{\lambda}_3(|z| + z_3) \theta(L - |z|) \\
b(z) &= \frac{y + x}{4} - \tilde{\lambda}_4(|z| + z_3) \theta(L - |z|) \\
\phi(z) &= \frac{x - y}{4} - 2\tilde{\lambda}_3(|z| + z_3) \theta(L - |z|) \\
\end{aligned} \tag{3.7}$$

where $\theta(z)$ is the Heaviside step function:

$$\theta(z) = \begin{cases} 1 & z > 0 \\ 0 & z < 0 \end{cases}$$

The solutions are continuous at $z = 0, \pm L$ and we demand them to be periodic with respect to $2L$ shift.

The first condition of (3.6) gives the following constraint,

$$3\lambda \coth(\lambda z_1) = 14\tilde{\lambda}_3 + 6\tilde{\lambda}_4 \tag{3.8}$$

from which together with (2.14) we obtain two constants $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$,

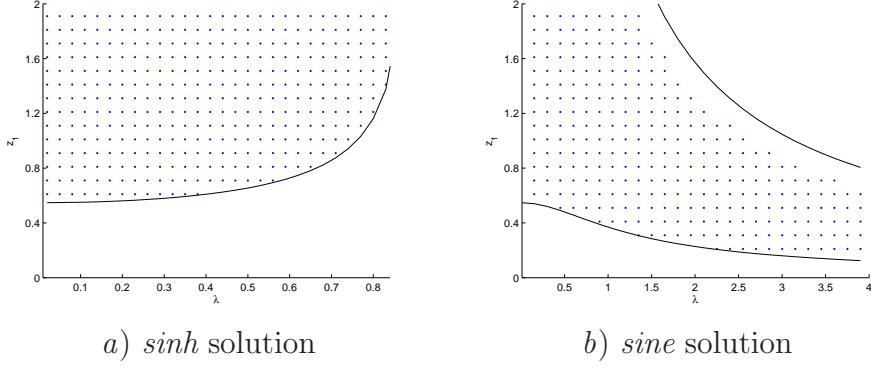


Figure 1: The dotted regions are permitted values of λ and z_1 for which we have real parameters $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$. In *a* the region is asymptote to maximum λ at $\sqrt{10/13}$. In *b* the upper curve shows an upper bound as $z_1 < \pi/\lambda$. Considering finite L the region $z_1 < \pi/\lambda - L$ gets smaller.

$$\tilde{\lambda}_3^\pm = \frac{3}{20}c\lambda \pm \frac{3}{40}\sqrt{-6c^2\lambda^2 - 20\lambda^2 + 20} \quad (3.9)$$

$$\tilde{\lambda}_4^\pm = \frac{3}{20}c\lambda \mp \frac{7}{40}\sqrt{-6c^2\lambda^2 - 20\lambda^2 + 20} \quad (3.10)$$

where $c = \coth \lambda z_1$ and the reality condition for $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ imposes the following inequality,

$$z_1 \geq \frac{1}{\lambda} \log \sqrt{k + \sqrt{k^2 - 1}} \quad (3.11)$$

where $k = (10 - 7\lambda^2)/(10 - 13\lambda^2)$.

Similarly for *sine* and *linear* solutions where $\lambda \rightarrow i\lambda$ and $\lambda \rightarrow 0$, respectively, we find the following regions in $\lambda - z_1$ plane:

$$\begin{aligned} z_1 &\geq \frac{1}{\lambda} \sin^{-1} \left(\sqrt{\frac{3\lambda^2}{13\lambda^2 + 10}} \right) && \text{sine solution} \\ z_1 &\geq \sqrt{\frac{3}{10}} && \text{linear solution} \end{aligned}$$

For the *sine* case we require that *sine* to be positive which gives $0 < \lambda(z + z_1) < \pi$, thus $(L + z_1) < \pi/\lambda$. The permitted regions in $\lambda - z_1$ plane are drown in figure 1.

From the other five junction conditions in (3.6) we derive the brane tensions,

$$\begin{aligned}
\kappa^2 T &= \left(\frac{8}{3} \tilde{\lambda}_3 + 2 \tanh(z_2) \right) e^{-v(0)} \\
\kappa^2 T_4 &= \left(\frac{20}{3} \tilde{\lambda}_3 + 4 \tilde{\lambda}_4 + 2 \tanh(z_2) \right) e^{-v(0)} \\
\kappa^2 T_{L0} &= \left(\frac{14}{3} \tilde{\lambda}_3 + 2 \tilde{\lambda}_4 - \lambda \coth(\lambda(L + z_1)) \right) e^{-v(L)} \\
\kappa^2 T_{L4} &= \left(-2 \tilde{\lambda}_3 - 2 \tilde{\lambda}_4 - \lambda \coth(\lambda(L + z_1)) + 2 \tanh(L - z_2) \right) e^{-v(L)} \\
\kappa^2 \tilde{T}_{L3} &= \left(-\frac{2}{3} \tilde{\lambda}_3 + 2 \tilde{\lambda}_4 + \lambda \coth(\lambda(L + z_1)) \right) e^{-v(L)}
\end{aligned} \tag{3.12}$$

where

$$\begin{aligned}
e^{-4v(0)} &= \frac{\tilde{g}^5 \lambda}{\kappa^6 \tilde{q}} \frac{\cosh^5(z_2)}{\sinh(\lambda z_1)} e^{-4\tilde{\lambda}_3 z_3} \\
e^{-4v(L)} &= \frac{\tilde{g}^5 \lambda}{\kappa^6 \tilde{q}} \frac{\cosh^5(L - z_2)}{\sinh(\lambda(L + z_1))} e^{-4\tilde{\lambda}_3(L + z_3)}
\end{aligned} \tag{3.13}$$

Notice that the brane tensions could be positive or negative depending on the parameters involved (λ , z_1 , z_2 and L). We may realize that we are living at $z = 0$ with an isotropic brane extension along our 4-dimensional space as in (3.4). So the relevant brane tension to us would be: T where its sign depends on λ , z_1 and z_2 . In figure 2, for one special value of z_2 , the positive and negative tension regions are shown for *sinh* and *sine* solutions, in $\lambda - z_1$ plane. The positive and negative regions shrink or expand by changing the value of z_2 .

Similar joining process should be considered for H field. The Maxwell equation (2.4) indicates that h'' is regular everywhere, on the other hand in (2.8), h' field solution admits both plus and minus signs. Thus it should change sign while crossing $z = 0$ and $z = L$. Therefore we take the plus sign for $0 < z < L$ and minus for $-L < z < 0$. Precisely at $z = 0$ and $z = L$ we take H to be zero. This implies vanishing H at $z = 0$ where is interpreted as the position of our 4-dimensional universe.

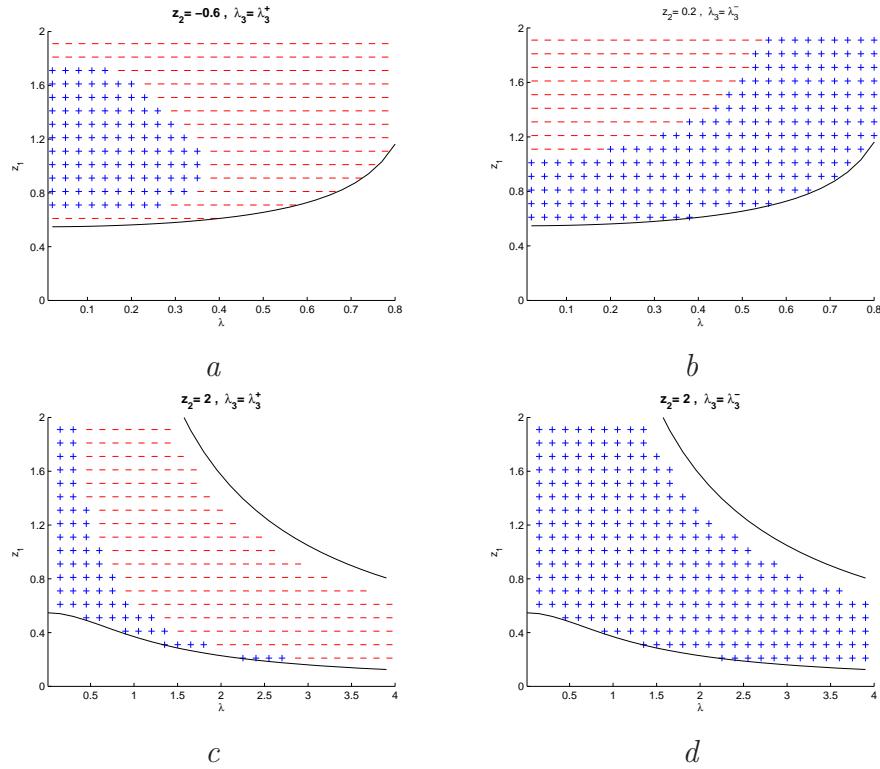


Figure 2: The plus and minus signs correspond to positive and negative tension T regions, respectively. Empty places are non-real tensions (non-real λ_3). The plots a, b are for \sinh and c, d are for \sin cases respectively.

4 Large L limit

Let us before studying the noncompact limit by sending L to infinity, introduce the proper radius ρ as

$$\rho = \int_0^\infty e^{v(z)} dz \quad (4.1)$$

then the internal 2-dimensional metric reads as

$$ds_2^2 = d\rho^2 + R^2(\rho)d\theta^2 \quad (4.2)$$

where $R(\rho) = e^{b(z(\rho))}$. Using numerical integration of (4.1), the shape of internal space is drawn for various amounts of parameters in figure 3 for the *sinh* case. Notice that the edges at $z = 0$ and $z = L$ are the places of branes. These are almost all possibilities that happen in the *sinh* case, either in the finite L or large L limit. In the rest we just concentrate on the *sinh* case. For *sine* case the upper limit, $L + z_1 < \pi/\lambda$, forbids the large L limit.

Beside this numerical integration, it is worth to study the behaviors of tensions and radius of the internal space for large L limit. Firstly, for brane tensions, the results in the previous section show that the branes at $z = 0$ are untouched when L is going to infinity. Thus we investigate branes sitting at L for very large L .

The brane tensions at large L are

$$\begin{aligned} \kappa^2 T_{L0}|_\infty &= \left(\frac{14}{3} \tilde{\lambda}_3 + 2\tilde{\lambda}_4 - \lambda \right) e^{-v} \\ \kappa^2 T_{L4}|_\infty &= \left(-2\tilde{\lambda}_3 - 2\tilde{\lambda}_4 - \lambda + 2 \right) e^{-v} \\ \kappa^2 \tilde{T}_{L3}|_\infty &= \left(-\frac{2}{3} \tilde{\lambda}_3 + 2\tilde{\lambda}_4 + \lambda \right) e^{-v} \end{aligned} \quad (4.3)$$

where e^{-v} for large L is

$$e^{-v} \sim Ae^{-\alpha L} \quad (4.4)$$

with $\alpha = (\lambda/4 + \tilde{\lambda}_3 - 5/4)$ and A is an L independent positive constant. From equations (3.9), we know that α is always negative. Thus all tensions goes to infinity at asymptotic distances.

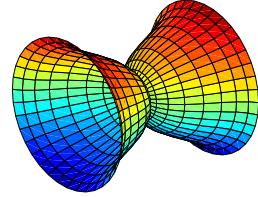
Now look at (4.4), ρ can be found for large z as,

$$\rho \sim \frac{1}{A} \int_0^z e^{\alpha z} dz = \frac{1}{A\alpha} (e^{\alpha z} - 1) . \quad (4.5)$$

Since α is negative, as z goes to infinity ρ approaches to $-1/(A\alpha)$ and for the radius we have,

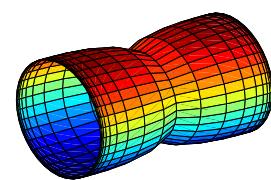
$$R_L \sim (1 + A\alpha\rho)^{-\frac{\beta}{\alpha}} \quad (4.6)$$

where $\beta = (\tilde{\lambda}_4 + \lambda/4 + 1/4)$. Thus as z goes to infinity, for negative β , R_L diverges and we have a noncompact space, while for nonnegative β the radius approaches to zero and a compact space is obtained (see figures 3, 4).

Finite L 

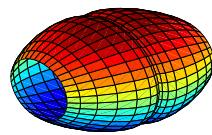
$$\lambda=0.4, z_1=1.5, z_2=1.5, \lambda_3=\lambda_3^+$$

(a)

Large L 

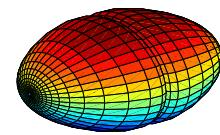
$$\lambda=0.59, z_1=1.5, z_2=0.5, \lambda_3=\lambda_3^+$$

(b)



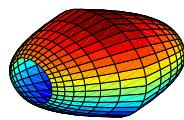
$$\lambda=0.8, z_1=2, z_2=1.5, \lambda_3=\lambda_3^+$$

(c)



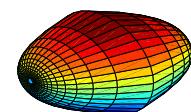
$$\lambda=0.8, z_1=1.5, z_2=2, \lambda_3=\lambda_3^+$$

(d)



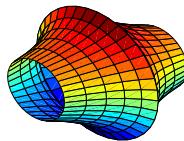
$$\lambda=0.8, z_1=2, z_2=-5, \lambda_3=\lambda_3^+$$

(e)



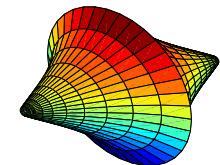
$$\lambda=0.8, z_1=2, z_2=-5, \lambda_3=\lambda_3^+$$

(f)



$$\lambda=0.3, z_1=1, z_2=1.5, \lambda_3=\lambda_3^-$$

(g)



$$\lambda=0.3, z_1=1, z_2=0.6, \lambda_3=\lambda_3^-$$

(h)

Figure 3: The shapes of internal space for various parameters . The axial direction is the ρ -axis. $\beta < 0$ for (a), $\beta = 0$ for (b) and $\beta > 0$ for others.

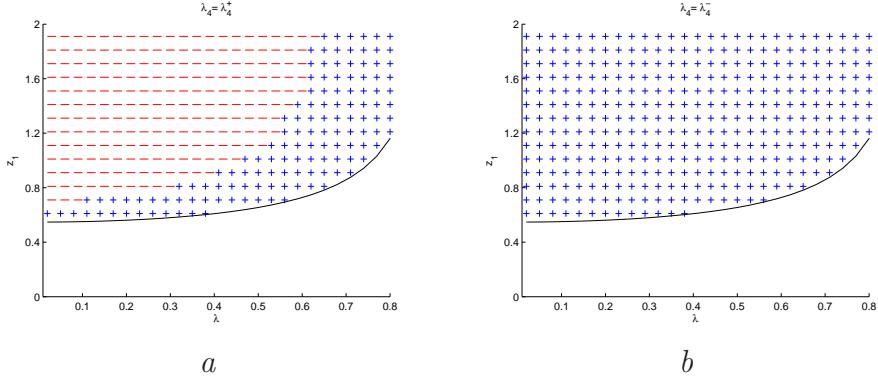


Figure 4: The plus and minus signs correspond to positive and negative β regions in the \sinh case, respectively.

5 Time warp consideration

Let us look at the null energy condition which can be stated as follows [24],

$$\tilde{T}_{MN}\xi^M\xi^N \geq 0 \quad (5.1)$$

where \tilde{T}_{MN} is constructed from energy-momentum tensor as $\tilde{T}_{MN} = T_{MN} - \frac{1}{d-2}g_{MN}T_L^L$ in a d -dimensional space. Then by the Einstein equation it leads to,

$$R_{MN}\xi^M\xi^N \geq 0 \quad (5.2)$$

for any time-like or null vector ξ^M .

Before checking out this condition in our case, we remind a related no-go theorem in [24], which states that for a class of solutions named ‘time warp’ the null energy condition can not be satisfied for compact extra dimensions. The time warp solutions are introduced as,

$$ds_d^2 = e^{2A(y)}[-h(y)dt^2 + d\bar{x}^2] + e^{2B(y)}d\bar{s}_{d-4}^2 \quad (5.3)$$

where y denotes the compact coordinates. The above metric covers our solution with $A = a$ and $h = \exp(2w - 2a)$. With this ansatz, the null energy condition gives,

$$4h^2e^{2B}(-R_0^0 + R_1^1) = -3\tilde{g}^{mn}\partial_mh\partial_nh + \tilde{\square}(h^2) + h\tilde{g}^{mn}\partial_mh\partial_n(8A + 2(d-6)B) \geq 0 \quad (5.4)$$

where m and n are extra directions indices. For $d \neq 6$ one can set $B = \frac{4}{6-d}A$ using the gauge freedom in y coordinate, then,

$$-3\tilde{g}^{mn}\partial_mh\partial_nh + \tilde{\square}(h^2) \geq 0 \quad (5.5)$$

Integrating over the compact extra dimensions implies h to be a constant.

Notice that this argument is valid only for $d \neq 6$. For our metric in (2.3) which is in $d = 6$ we find,

$$e^{2v}(-R_0^0 + R_1^1) = w'' - a'' + (w' - a')^2 + (b' - v')(w' - a') + 4a'(w' - a') \geq 0 \quad (5.6)$$

which can be converted to the form of (5.4) for $d = 6$ with $h = \exp(2w - 2a)$ and $b = v$. We have already chosen a gauge freedom in (2.7) by which we can escape the no-go theorem. Plugging (2.7) in the above inequality one finds the following simple constraint,

$$w'' - a'' \geq 0 \quad (5.7)$$

On the other hand,

$$\begin{aligned} w'' - a'' &= x'' \\ &= \lambda^2 (-1 + \coth^2(\lambda(z + z_1))) \geq 0 \end{aligned} \quad (5.8)$$

which is always true (for *sin* and *linear* case one can send λ to $i\lambda$ and zero, respectively, which both satisfy the inequality). This shows that we have constructed a solution which satisfies the energy constraint and escapes the no-go theorem, even in the compact case. There is no contradiction here, since the no-go theorem is valid for $d \neq 6$ and we have a counterexample for $d = 6$.

6 Effective 4-dimensional Planck mass

In the usual extra dimensional theories, effective 4D theory is obtained via integrating over the extra dimensions and interpreting the higher dimensional M-Planck multiplied by the volume of extra dimension as the effective 4D M-Planck. However, the warp factor of time is different from the warp factor of space in here, so we should change the usual procedure.

Let us decompose the 6-dimensional Ricci scalar to the 4-dimensional one in the action as,

$$\begin{aligned} S_6 &= M_{(6)}^4 \int \sqrt{-G} R^{(6)} d^6x \\ &= M_{(6)}^4 \int d^4x \sqrt{-g} \left(-R_{00}^{(4)} \int d\theta d\eta \sqrt{G} e^{-2w} + \delta^{ij} R_{ij}^{(4)} \int d\theta d\eta \sqrt{G} e^{-2a} \right) \end{aligned} \quad (6.1)$$

where $R^{(6)}$ is the 6D Ricci scalar, g is the determinant of the flat metric of 4D theory, G is the determinant of 6D theory and the 6D Planck-mass is, $M_{(6)}^4 = \frac{1}{2\kappa^2}$. We require that:

$$\int d\theta d\eta \sqrt{G} e^{-2w} = \int d\theta d\eta \sqrt{G} e^{-2a} =: V \quad (6.2)$$

where the integration is over the range of η . Now we define the 4D Planck-Mass as:

$$M_{(4)}^2 = \frac{1}{\kappa^2} V \quad (6.3)$$

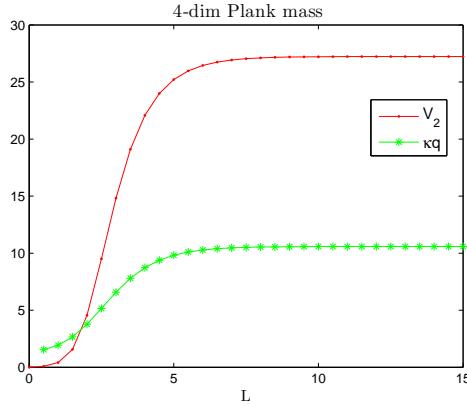


Figure 5: The planck mass $M_P = \frac{L_\theta}{\lambda_2 \tilde{g}^2} V_2$ and $\kappa \tilde{q}$ as functions of L , for $\lambda = 0.3$, $z_1 = 0.7$ and $z_2 = 2$.

$$\begin{aligned}
\int d\theta d\eta \sqrt{G} e^{-2w} &= \int d\theta d\eta e^{2y-x} e^{-2(\lambda_3 + \lambda_4)\eta} \\
&= L_\theta \frac{\kappa^3 \tilde{q}}{\lambda_1 \tilde{g}^2} \int_{-L}^L dz \frac{\sinh(\lambda(|z| + z_1))}{\cosh^2(|z| - z_2)} e^{-2(\tilde{\lambda}_3 + \tilde{\lambda}_4)|z|} \\
&=: L_\theta \frac{\kappa^3 \tilde{q}}{\lambda_1 \tilde{g}^2} V_1
\end{aligned} \tag{6.4}$$

$$\begin{aligned}
\int d\theta d\eta \sqrt{G} e^{-2a} &= \int d\theta d\eta e^{2y} e^{\frac{8}{3}\lambda_3\eta} \\
&= L_\theta \frac{\kappa^2}{\lambda_2 \tilde{g}^2} \int_{-L}^L dz \frac{e^{\frac{8}{3}\tilde{\lambda}_3|z|}}{\cosh^2(|z| - z_2)} \\
&=: L_\theta \frac{\kappa^2}{\lambda_2 \tilde{g}^2} V_2
\end{aligned} \tag{6.5}$$

Equating (6.4) and (6.5) fixes one parameter say $\kappa \tilde{q}$,

$$\kappa \tilde{q} = \lambda \frac{V_2}{V_1} \tag{6.6}$$

Finally we find the effective 4-dimensional theory as,

$$S^{(4)} = \frac{1}{\kappa^2} V \int d^4x \sqrt{-g} R^{(4)} \tag{6.7}$$

with V given in (6.5).

Notice that our model starts with an asymmetrical spacetime due to the presence of the 3-form field H , however at the end by a fine tuning of \tilde{q} which is the charge of H , one can reach to an effective 4-dimensional symmetric gravity.

It is worth to consider the large L limit which correspond to the case that the extra dimension is not compact and the branes at L are sending to infinity. The integrals in (6.4) and (6.5) remain finite for $L \rightarrow \infty$ which gives us a finite effective 4-dimensional Planck mass (see figure 5).

7 Conclusion

We have solved the static equations of motion for 6-dimensional Salam-Sezgin model in the presence of 3-form field H which provides a 4-dimensional compactification. To find out a global solution over the compact manifold, we consider different patches and join them with the Israel junction conditions which can be satisfied with inserting some branes at the junctions. These conditions also fix some integration constants. More explicitly, we have considered the compact space with angular coordinate θ , and radial coordinate z where the space is defined to be periodic with fundamental region $z \in [-L, L]$ and even under $z \rightarrow -z$. This gives the torus topology. Then to satisfy the Israel conditions, 3 and 4 branes are inserted at $z = 0$ such that they are extended along our 4 dimensional space-time and 4 brane wrapped and 3 branes smeared over the θ circle. The situation is the same at $z = L$ except that we need to add some 0-branes smeared over the 4 dimensional worldvolume of the 4-brane. We may consider $z = 0$ where our brane-universe sits.

We have studied the solution behaviors in different regions of independent parameters and specially for large L limit we found that in some cases the internal radius of θ circle shrinks and changes the topology from torus to sphere.

The asymmetry in space and time is due to the presence of the H field. This kind of warping with different time and space warp factors are recently studied in [24] and called ‘time warp’ compactification. It is known that this compactification violates the null energy condition in $d \neq 6$ dimensions [24]. However our compactification which is of course for $d = 6$, shows that the null energy condition is satisfied with a time warp compact space. In section 5, We tried to show why this happens.

Our branes configuration makes it possible to have a 4-dimensional symmetric space at $z = 0$. This can be supplemented with the fact that $H = 0$ at $z = 0$. Indeed H is discontinuous at this position, and changes the sign while crossing the brane. The mean value of H would be zero. There is another view in which the H field exponentially vanishing at the other end, $z = L$, for very large L . This enables us to reverse the situation by putting 0-branes at $z = 0$ and find a symmetric space-time at $z = L$ for large L where H vanishes and branes preserve the lorentz symmetry.

Another important issue is introducing an effective 4-dim Planck mass. We have done it by firstly expanding the 6-dimensional gravity action and then integrate out the extra dimensions. Since the solution has two different warp factors for time and space, we encounter with two different integrations. Equating these two integrals we fix the charge q of the H field and we can factor out integrals over the internal space and find the 4-dimensional Planck mass.

This model is restricted to a static solution, the next development should be a dynamic solution in which all fields would be time dependent. This is consistent with the presence of H and would be important if one is interested in finding cosmological application of this model. The stability of this model should be checked and may stabilize some parameters (work in progress).

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